

Collapsing thin shells with rotation

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We construct exact solutions describing the motion of rotating thin shells in a fully backreacted five-dimensional rotating black hole spacetime. The radial equation of motion follows from the Darmois-Israel junction conditions, where both interior and exterior geometries are taken to be equal angular momenta Myers-Perry solutions. We show that rotation generates anisotropic pressures and momentum along the shell. Gravitational collapse scenarios including rotation are analyzed and a new class of stationary solutions is introduced. Energy conditions for the matter shell are briefly discussed.

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Introduction. — Dynamical processes in rotating spacetimes are notoriously difficult to model analytically. In fact, there exist very few exact solutions in closed form describing collapsing matter carrying angular momentum. A few notable exceptions were obtained in 2+1 dimensions, where the problem naturally becomes more tractable [1, 2] (see [3, 4] for four-dimensional, but cylindrically symmetric, studies). The aim of this Letter is to construct exact rotating solutions to the Einstein field equations in five spacetime dimensions involving matter shells around black holes. In particular, these solutions allow to address for the first time the effect of rotation on the gravitational collapse in an exact manner in a scenario with more than two spatial dimensions.

Our setup consists of a gravitating thin rotating shell in five dimensions, which backreacts on the geometry of a black hole. We allow for the presence of a cosmological constant. Our construction relies on the well known Darmois-Israel formalism [5, 6] for matching two spacetimes along a spacelike hypersurface. This framework was originally employed in [7] to study the spherical collapse of thin dust rings in 2+1 dimensions. More recently it was extended to the rotating case and to shells with pressure in Ref. [1] (see [8] for an early study in four dimensions employing a slow rotation approximation).

The generalization to higher dimensions is not straightforward at all, the reason being that the spacetime will typically depend non trivially on polar angles in addition to a radial coordinate, thus rendering the matching procedure intractable on analytic grounds. However, it is possible to make progress by restricting to the class of higher (odd) dimensional black holes with all — generically independent — angular momenta equal: in this case the problem is cohomogeneity-1 [9–11]. For simplicity we develop the formalism in 4+1 dimensions but similar re-

sults are expected for higher odd dimensions.

Throughout the manuscript we use geometrized units, by setting both the speed of light c and Newton’s constant G equal to one.

Equally spinning black holes in five dimensions. — The rotating black hole geometry generalizing the Kerr solution to higher dimensions is the well known Myers-Perry black hole [12], which has been extended to include a cosmological constant in [13, 14].

Five-dimensional spacetimes admit two orthogonal rotation planes, and consequently two independent angular momenta. When the two angular momenta are equal, the isometry group of the solution gets enhanced and the metric functions can be written in terms of a single coordinate r [11]:

$$ds^2 = -f(r)^2 dt^2 + g(r)^2 dr^2 + r^2 \hat{g}_{ab} dx^a dx^b + h(r)^2 [d\psi + A_a dx^a - \Omega(r) dt]^2, \quad (1)$$

where

$$g(r)^2 = \left(1 + \frac{r^2}{\ell^2} - \frac{2M\Xi}{r^2} + \frac{2Ma^2}{r^4} \right)^{-1}, \quad (2)$$

$$h(r)^2 = r^2 \left(1 + \frac{2Ma^2}{r^4} \right), \quad \Omega(r) = \frac{2Ma}{r^2 h(r)^2}, \quad (3)$$

$$f(r) = \frac{r}{g(r)h(r)}, \quad \Xi = 1 - \frac{a^2}{\ell^2}, \quad (4)$$

and with \hat{g}_{ab} and $A = A_a dx^a$ given by

$$\hat{g}_{ab} dx^a dx^b = \frac{1}{4} (d\theta^2 + \sin^2 \theta d\phi^2), \quad A = \frac{1}{2} \cos \theta d\phi. \quad (5)$$

The line element in the form (1) generalizes to higher odd dimensions $D = 2N + 3$, with N an integer [11]. Here, we shall restrict to the case $N = 1$, for which the

metric (1) is a solution of the vacuum Einstein equations with a cosmological constant, equal to $\Lambda = -6\ell^{-2}$. The asymptotically flat case can be recovered by taking the limit $\ell \rightarrow \infty$.

The largest real root of g^{-2} marks an event horizon which possesses the geometry of a homogeneously squashed S^3 . The mass \mathcal{M} and angular momentum \mathcal{J} of the spacetime are given by [11]

$$\mathcal{M} = \frac{\pi M}{4} \left(3 + \frac{a^2}{\ell^2} \right), \quad \mathcal{J} = \pi M a. \quad (6)$$

Matching spacetimes. — We take two spacetimes with line element given by Eq. (1) for the interior and exterior spacetime. This metric describes a family of black hole spacetimes, parametrised by the pair (M, a) . We further add an index α to these parameters since in general the inner and outer spacetimes have different parameters. We take $\alpha = +$ ($\alpha = -$) for the exterior (interior) geometry. Similarly, the metric functions in (2-4) also acquire an index α , reflecting the choice of mass and spin parameters.

Let $\Sigma = \{x^\mu : t = \mathcal{T}(\tau), r = \mathcal{R}(\tau)\}$ be the hypersurface along which the shell lies. This 4D surface can be parametrised by coordinates $y^i = \{\tau, \psi, \theta, \phi\}$. Denote by $\mathfrak{g}_{ij}^{(\alpha)}$ the induced metric on Σ as determined from the exterior/interior solution, and by $k_{ij}^{(\alpha)}$ the associated extrinsic curvature, with $k^{(\alpha)}$ being its trace. The Darmois-Israel junction conditions can be expressed as

$$\mathfrak{g}_{ij}^{(+)} = \mathfrak{g}_{ij}^{(-)} \equiv \mathfrak{g}_{ij}, \quad (7)$$

$$(k_{ij}^{(+)} - k_{ij}^{(-)}) - \mathfrak{g}_{ij}(k^{(+)} - k^{(-)}) = -8\pi G \mathcal{S}_{ij}, \quad (8)$$

where \mathcal{S}_{ij} stands for the surface stress-energy tensor.

Before computing the junction conditions it is useful [1] to convert to the comoving frame to eliminate cross terms in the induced metrics, by making the following change of coordinates

$$d\psi \longrightarrow d\psi' + \Omega_\alpha(\mathcal{R}(t))dt. \quad (9)$$

The exterior and interior metrics then become

$$ds_\alpha^2 = -f_\alpha(r)^2 dt^2 + g_\alpha(r)^2 dr^2 + r^2 \hat{g}_{ab} dx^a dx^b + h_\alpha(r)^2 [d\psi' + A_\alpha dx^a + (\Omega_\alpha(\mathcal{R}(t)) - \Omega_\alpha(r))dt]^2. \quad (10)$$

From now on we will drop the prime in ψ' and assume we are in the comoving frame, in which case $\mathfrak{g}_{\tau j} = -\delta_{\tau j}$.

The first junction condition (7) implies that the combination Ma^2 is invariant across the shell [15] and also provides a relation between $(\frac{dT}{d\tau})^2$ and $(\frac{d\mathcal{R}}{d\tau})^2$:

$$M_+ a_+^2 = M_- a_-^2, \quad (11)$$

$$-f_\alpha(\mathcal{R})^2 \left(\frac{dT}{d\tau} \right)^2 + g_\alpha(\mathcal{R})^2 \left(\frac{d\mathcal{R}}{d\tau} \right)^2 = -1. \quad (12)$$

The extrinsic curvature is obtained from $k_{\mu\nu} = (g_{\mu\sigma} - n_\mu n_\sigma) \nabla^\sigma n_\nu$, where $n_\mu = f(r)g(r) \left(-\frac{d\mathcal{R}}{d\tau}, \frac{dT}{d\tau}, 0, 0 \right)$, is the unit normal vector to the hypersurface Σ . In terms of the coordinates y^i along the hypersurface, the induced extrinsic curvature is written as $k_{ij} = k_{\mu\nu} \frac{dx^\mu}{dy^i} \frac{dx^\nu}{dy^j}$.

The second junction condition (8) requires the shell stress-energy tensor to take the form of an imperfect fluid with anisotropic pressure and intrinsic momentum:

$$\mathcal{S}_{ij} = (\rho + P)u_i u_j + P \mathfrak{g}_{ij} + 2\varphi u_{(i} \xi_{j)} + \Delta P \mathcal{R}^2 \hat{g}_{ij}, \quad (13)$$

where $u = \partial_\tau$ is the (normalized) fluid four-velocity, $\xi = h(\mathcal{R})^{-1} \partial_\psi$ and $\hat{g}_{ij} dy^i dy^j = \hat{g}_{ab} dx^a dx^b$. Collapse with fluids of this type was recently considered in [16]. Observe that when $\Delta P = \varphi = 0$ we retrieve the stress-energy tensor of a perfect fluid [17].

We find that the junction conditions (8) are satisfied if and only if

$$\rho = -\frac{(\beta_+ - \beta_-)(\mathcal{R}^2 h)'}{8\pi \mathcal{R}^3}, \quad \varphi = -\frac{(\mathcal{J}_+ - \mathcal{J}_-)(\mathcal{R} h)'}{4\pi^2 \mathcal{R}^4 h}, \quad (14)$$

$$P = \frac{h}{8\pi \mathcal{R}^3} [\mathcal{R}^2(\beta_+ - \beta_-)]', \quad \Delta P = \frac{(\beta_+ - \beta_-)}{8\pi} \left[\frac{h}{\mathcal{R}} \right]',$$

where we have introduced the quantities

$$\beta_\alpha \equiv f_\alpha(\mathcal{R}) \sqrt{1 + g_\alpha(\mathcal{R})^2 \left(\frac{d\mathcal{R}}{d\tau} \right)^2}, \quad (15)$$

and where primes stand for $d/d\mathcal{R}$. The constraint (11) is already being used, so that $h_+(\mathcal{R}) = h_-(\mathcal{R}) \equiv h(\mathcal{R})$ and $h'_+(\mathcal{R}) = h'_-(\mathcal{R}) \equiv h'(\mathcal{R})$. Note that the momentum φ and the anisotropic pressure term ΔP necessarily vanish in the limit of zero rotation. We stress that the component φ controls the difference between the angular momentum of the outer and inner spacetimes.

The strategy we have adopted, relying on the Darmois-Israel matching formalism, improves on the perturbative approach developed in Ref. [18], in the sense that the solutions constructed herein account for all backreaction effects. In [18] it was found that the shell was required to be corotating with the spacetime [19] but this can now be relaxed at the expense of the fluid acquiring intrinsic momentum and anisotropic pressure.

Energy conditions. — Energy conditions for imperfect fluids, such as (13), have been studied in [20] and are most easily formulated in terms of the eigenvalues of the stress-energy tensor, say λ_0, λ_i , ($i = 1, 2, 3$), where λ_0 is the eigenvalue associated with the time-like eigenvector. The weak energy condition (WEC) reads [21]

$$-\lambda_0 \geq 0, \quad \lambda_i - \lambda_0 \geq 0, \quad (16)$$

while the less restrictive null energy condition simply omits the first inequality.

In the specific case of the fluid (13), the eigenvalues are given by

$$\begin{aligned}\lambda_0 &= \frac{P - \rho}{2} - \sqrt{\left(\frac{P + \rho}{2}\right)^2 - \varphi^2}, \\ \lambda_1 &= \frac{P - \rho}{2} + \sqrt{\left(\frac{P + \rho}{2}\right)^2 - \varphi^2}, \\ \lambda_2 &= \lambda_3 = P + \Delta P.\end{aligned}\quad (17)$$

This must be supplemented with the constraint $\rho + P \geq 0$, ensuring that the eigenvector corresponding to λ_0 is time-like [22].

Shell equation of motion. — Let us assume a linear equation of state (EoS) of the form $P = w\rho$. Using Eqs. (14) this EoS translates into

$$\frac{[\mathcal{R}^2(\beta_+ - \beta_-)]'}{\mathcal{R}^2(\beta_+ - \beta_-)} = -w \frac{[\mathcal{R}^2 h]'}{\mathcal{R}^2 h}, \quad (18)$$

which admits the general solution

$$\beta_+ - \beta_- = -\frac{m_0^{1+3w/2}}{\mathcal{R}^{2(1+w)} h(\mathcal{R})^w}. \quad (19)$$

Here, m_0 is a positive constant with dimensions of mass. For the case of a non-rotating shell of dust ($a, w = 0$) it corresponds to the initial rest mass of the shell.

Employing expression (15), this solution provides an equation for the radial motion of the shell,

$$\dot{\mathcal{R}}^2 + V_{\text{eff}}(\mathcal{R}) = 0, \quad (20)$$

where $\dot{\mathcal{R}} \equiv d\mathcal{R}/d\tau$ and the effective potential V_{eff} is given by

$$\begin{aligned}V_{\text{eff}}(\mathcal{R}) &= 1 + \frac{\mathcal{R}^2}{\ell^2} + \frac{2Ma^2}{\mathcal{R}^4} + \frac{2Ma^2}{\ell^2 \mathcal{R}^2} - \frac{M_+ + M_-}{\mathcal{R}^2} \\ &\quad - \left(\frac{M_+ - M_-}{m_0}\right)^2 \left(\frac{\mathcal{R}^2}{m_0}\right)^{3w} \left(1 + \frac{2Ma^2}{\mathcal{R}^4}\right)^{w-1} \\ &\quad - \frac{1}{4} \left(\frac{m_0}{\mathcal{R}^2}\right)^{2+3w} \left(1 + \frac{2Ma^2}{\mathcal{R}^4}\right)^{1-w}.\end{aligned}\quad (21)$$

Given a set of parameters, namely the inverse curvature scale ℓ (including the flat limit $\ell \rightarrow \infty$), the interior and exterior mass parameters, M_+ , M_- , the value of Ma^2 and the fluid parameters w and m_0 , a solution describing the collapse can be found by solving (20).

First, let us examine the asymptotic behavior of the potential. For large values of \mathcal{R} , it reads

$$V_{\text{eff}} \approx 1 + \frac{\mathcal{R}^2}{\ell^2} - \left(\frac{\Delta M}{m_0}\right)^2 \left(\frac{\mathcal{R}^2}{m_0}\right)^{3w} - \frac{1}{4} \left(\frac{m_0}{\mathcal{R}^2}\right)^{2+3w}, \quad (22)$$

while for small radii it reduces to

$$\begin{aligned}V_{\text{eff}} &\approx \frac{2Ma^2}{\mathcal{R}^4} - \frac{M_+ + M_-}{\mathcal{R}^2} - \frac{1}{4} \left(\frac{2Ma^2}{m_0^2}\right)^{1-w} \left(\frac{m_0}{\mathcal{R}^2}\right)^{4+w} \\ &\quad - \left(\frac{2Ma^2}{m_0^2}\right)^{w-1} \left(\frac{\Delta M}{m_0}\right)^2 \left(\frac{\mathcal{R}^2}{m_0}\right)^{2+w},\end{aligned}\quad (23)$$

where we have defined $\Delta M \equiv M_+ - M_-$. (We have kept the subleading terms that dominate in the flat or nonrotating limits.) For $w = 0$, we recover the AdS barrier at large radius (for positive ℓ^{-2}) and the centrifugal repulsion term $\sim \mathcal{R}^{-4}$ at small radius. However, observe that the confining potential is absent if $w > 1/3$ or $w < -1$, and the centrifugal repulsion is counteracted by a term $\sim \mathcal{R}^{-8-2w}$ when $w > -2$. We also note that in the flat limit, the only possibility to describe a collapse starting at rest from infinity occurs for the choice of dust ($w = 0$) and only if the condition $\Delta M = m_0$ is satisfied. For a such a collapse, the energy of the spacetime will be enhanced by the rest mass of the shell.

Full collapse onto a black hole. — As an illustration, we now provide an explicit example of a rotating dust thin shell collapsing from rest at infinity onto a (small) rotating black hole in an asymptotically Minkowski spacetime. This is to the best of our knowledge the first exact solution of this sort in more than three spacetime dimensions.

In accordance with the above results we now take $\ell \rightarrow \infty$, set $w = 0$ and $m_0 = \Delta M$. It is easy to find conditions on the remaining parameters $\{Ma^2, M_+, M_-\}$ so that the effective potential is strictly negative (corresponding to a full collapse) and the eigenvalues of the stress-energy tensor are real and nonnegative (required for the weak energy conditions to be satisfied). In Figure 1 we show these quantities as functions of the radial location of the shell. A convenient choice of parameters was made for illustration purposes but we stress that no fine tuning is necessary.

This dynamical solution should be thought of as follows. Initially, the shell is located at $\mathcal{R} \rightarrow \infty$ with vanishing radial velocity. It then collapses according to Eq. (20). The spacetime in the range $r < \mathcal{R}(\tau)$ is described by the inner geometry and consists of a rotating black hole with a horizon at $r = r_h^-$ and spin parameter $a_- = \sqrt{Ma^2/M_-}$. At larger radii, the spacetime is the exterior of a black hole with horizon radius r_h^+ and spin parameter $a_+ = \sqrt{Ma^2/M_+}$. As the shell crosses the radius r_h^+ , a new apparent horizon forms at that location. Both the mass and spin of the initial black hole have changed. Eventually the shell falls into the curvature singularity at $r = 0$, where the stress-energy components also blow up.

Note that none of the collapsing solutions we have described violate the cosmic censorship conjecture [23]: the identity (11) implies that if the interior solution is sub-extremal the global solution after full collapse is similarly sub-extremal.

Stationary solution in AdS. — Given the ‘confining’ nature of the effective potential in the presence of a negative cosmological constant (and when $-1 < w < 1/3$), as well as the existence of a centrifugal barrier, we expect that stationary solutions consisting of a rotating shell

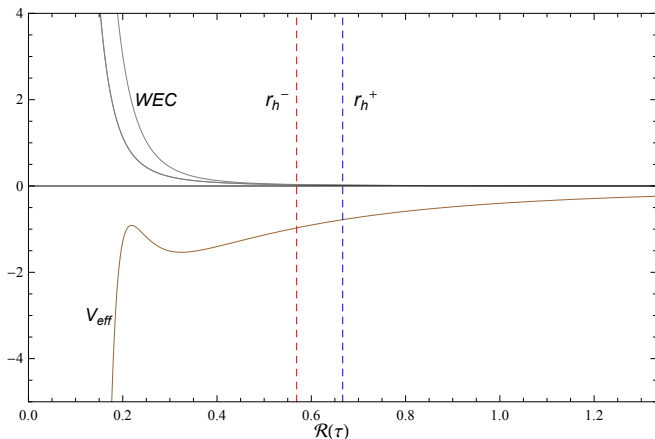


FIG. 1. Gravitational collapse of a rotating shell (with $w = 0$) in five-dimensional asymptotically flat spacetime. The functions $-\lambda_0$, $\lambda_1 - \lambda_0$, $\lambda_2 - \lambda_0$ are collectively denoted by WEC. The curves for $-\lambda_0$ and $\lambda_2 - \lambda_0$ nearly coincide for the solution displayed. The horizon radius of the inner (outer) geometry is denoted by r_h^- (r_h^+). Note that the WEC are satisfied throughout the collapse, all the way up to the singularity. This particular plot corresponds to parameters $M_- = 0.2$, $M_+ = 0.25$, $Ma^2 = 0.01236$, $m_0 = \Delta M = 0.05$ (in arbitrary units), which are chosen in such a way that the collapse is complete.

hovering around a black hole exist in AdS.

Such solutions can be constructed as follows. We assume that (i) the radial location of the shell is constant, i.e., $\mathcal{R}(\tau) = \mathcal{R}_*$, implying that $V_{\text{eff}}(\mathcal{R}_*) = 0$, and that (ii) it corresponds to a local minimum of the potential, $V'_{\text{eff}}(\mathcal{R}_*) = 0$ and $V''_{\text{eff}}(\mathcal{R}_*) > 0$. In general, these conditions involve high order polynomial equations in \mathcal{R}_* . On the other hand, the conditions are simple quadratic equations in M_+ and M_- (see Eq. (21)). Therefore, it is more convenient to fix the masses of the inner and outer space in terms of the remaining parameters, $\{w, m_0, \mathcal{R}_*, Ma^2\}$. This can be done in such a way to ensure that $\mathcal{R} = \mathcal{R}_*$ is a stable point. Then one just needs to scan the parameter space to obtain solutions for which the inner geometry possesses an event horizon, while the outer geometry has no horizons, and satisfy the WEC. It is easy to find such solutions and we show an example in Figure 2.

We mention in passing the existence of unstable stationary solutions in flat space, where the gravitational attraction is balanced by the centrifugal repulsion. These configurations are obtained by tuning the parameters so that the potential of Fig. 1 acquires a maximum with 0 value.

Discussion. — In this Letter we showed it is possible to address non spherical gravitational collapse with analytic tools and nonperturbatively, even without restricting to lower dimensional spacetimes. In particular, we developed the formalism to study the motion of rotating thin shells in equal angular momenta black hole spacetimes in five dimensions, allowing for a cosmologi-

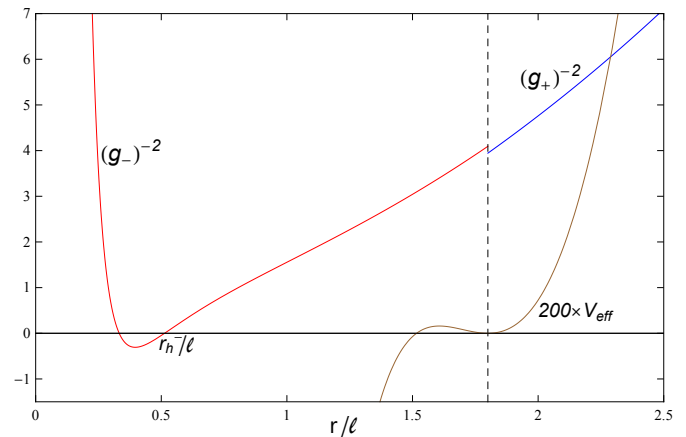


FIG. 2. Stable configuration of a stationary rotating shell in AdS. The dashed line represents the location \mathcal{R}_* of the shell. We plot the metric function $g(r)^{-2}$, whose roots mark horizons. In the domain $r > \mathcal{R}$ this function is given by $g_+(r)^{-2}$ (blue) and for $r < \mathcal{R}$ by $g_-(r)^{-2}$ (red). Notice the discontinuity at $r = \mathcal{R}_*$. The event horizon radius r_h^- is given by the largest real root of g_-^{-2} . The brown curve represents the effective radial potential (amplified to ease interpretation) and $r = \mathcal{R}_*$ corresponds to a local minimum of the potential. We chose the following values of parameters for this plot: $m_0/\ell^2 = 0.324$, $w = 0.285$, $Ma^2/\ell^4 = 0.02$, $\mathcal{R}_*/\ell = 1.8$.

cal constant.

The consideration of five dimensions is not physically fundamental: we regard it as a technical crutch to make progress analytically. We also stress that, while the symmetry required for the success of our analysis is enhanced relative to the case with arbitrary angular momenta ($U(2)$ compared to $U(1)^2$), it is still more generic than the spherically symmetric situation, which has isometry group $SO(4) \simeq SU(2) \times SU(2)$.

The most important lesson borne out by this work is that anisotropic pressures and intrinsic momentum are necessarily generated when rotation is present. This result is independent of the shell equation of state and is expected to hold even in less symmetric collapse scenarios. We further believe that the departure of the stress-energy tensor from the perfect fluid form is typical of collapse with rotation also in less idealized cases, in which the shell has finite thickness. In particular, we expect numerical simulations of gravitational collapse to display this feature.

We presented two examples of exact solutions: a full collapse with rotation onto an asymptotically flat black hole and a stationary configuration of matter around a rotating black hole in AdS. Our analysis of the parameter space of solutions is far from being exhaustive. Such an extension is planned to be studied elsewhere.

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